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Phenomenological Description of Particle and Entropy Creation

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Thermodynamic quantities and those describing the source of gravitational fields need not be identified. The standard approach is to identify them. If one does not follow this way, one opens the possibility of describing, on the level of phenomenology, the "average" of the quantum physical process behind the creation of particles and entropy in a comoving volume. Different approaches are briefly discussed. A variational principle with constraints modeling particle and entropy creation is formulated and leads to higher-order gravity field equations. They suggest on the level of phenomenology that the Minkowski space-time could be unstable against vacuum fluctuations.

1. INTRODUCTION

The question of what the source of gravitation is has been a subject of discussion as long as there has been a theory of gravitation. The standard point of view is that in a relativistic theory of gravitation the source term is given by the energy-momentum tensor of matter. But already here the problem is that one can have different ideas about what matter is. Take, for example, the scalar tensor theories of gravitation! There, roughly speaking, the source of gravitation can be the energy-momentum tensor of common matter and a tensor given by a scalar field. Now you can say matter is the "sum" of "common" matter and a scalar field or matter is only "common" matter. In the latter case, it will happen that matter is created or annihilated (Hoyle and Narlikar, 1963). The possibility that particles could be created was also discussed by Dirac (1974). The same subject was also discussed in Prigogine *et al.* (1988) and Prigogine (1989). There, starting point is the idea that thermodynamic quantities and those quantities describing the source of gravitation need not be identical.

No doubt, the elementary process of creating particles has to be the subject of a model on the level of quantum field theory. [For those who,

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like the author, are not working on quantum field theory in curved space-time, Fulling (1989) is an excellent reference.] Fulling (1989, p. 185) writes "that the phenomenon of particle creation by the gravitational field should be expected to disrupt the energy conservation law ... is unconvincing." And he refers to the creation of an electron–positron pair by external electromagnetic fields that "does not violate the electromagnetic charge-current conservation law." However, one must admit that both the mentioned conservation laws are not upon the same footing. At least on the level of classical physics, global quantities such as total energy and total momentum are generally not well defined. But let us suppose the divergence of the energy-momentum tensor operator is equal to zero on the level of quantum field theory. Is it really a consequence then that the divergence of an energy-momentum tensor describing the classical approximation (if existing) is equal to zero, too? Perhaps there is the possibility that "particles" with positive and negative energy such that the sum of both kinds of energy is equal to zero are created and the "particles" with negative energy are "pressed" into the vacuum. One could imagine that part of the vacuum is a "sea" of black holes absorbing the negative-energy "particles" via a process similar to the Penrose process (Penrose, 1969). To describe the remaining particles with positive energy, one had, at the level of a classical approximation, only the possibility to suppose that the divergence of the energy-momentum tensor is not equal to zero.

This idea is behind the final section of this paper. There, one result is that the Minkowski space-time is probably unstable with respect to fluctuations of a radiation field. In cosmology, it is argued sometimes that our universe is the result of a quantum vacuum fluctuation. Perhaps the mentioned result reflects this suggestion on the level of a classical approximation.

Section 2 presents some general remarks on the problem and Section 3 offers a phenomenological basis for so-called higher-order gravity.

2. ON ADIABATIC TRANSFORMATIONS OF OPEN SYSTEMS

On the level of a phenomenological description of matter we can distinguish thermodynamic and effective quantities, the latter determining, by definition, the source of gravitation. Let us consider, as an example, a simple fluid defined by an equation of state such that the thermodynamic energy density ρ is a given function of the particle number density n and the entropy density s. This fluid is assumed to fill a homogeneous and isotropic universe. We additionally assume that the space slices are flat. Then the line element reads, in spherical coordinates,

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)) \tag{1}$$

Table I

Thermodynamic quantities		Effective quantities		Geometric quantities	
ρ	energy density	$\bar{\rho}$	effective energy density	a	scale factor
n	particle number density	$\bar{p}$	effective pressure	H	Hubble parameter
p	pressure				
s	entropy density				
T	temperature				
μ	chemical potential				
s_a	entropy per particle				
dQ	change of heat contained in a volume				

We have the sets of quantities shown in Table I.

The question arises: How are these quantities related? They enter the first law of thermodynamics,

$$-\frac{dQ}{dt}+(\rho a^3)^{\cdot}+3Hpa^3-\frac{\rho+p}{n}(na^3)^{\cdot}=0 \tag{2}$$

Einstein's equations

$$H^2=\frac{\kappa}{3}\bar{\rho}, \qquad (\bar{\rho}a^3)^{\cdot}+\bar{p}(a^3)^{\cdot}=0 \tag{3}$$

and the second law of thermodynamics

$$(\rho a^3)^{\cdot}+p(a^3)^{\cdot}-T(sa^3)^{\cdot}-\mu(na^3)^{\cdot}=0 \tag{4}$$

The important point is that $(na^3)^{\cdot}\neq 0$ and $(sa^3)^{\cdot}\neq 0$ means creation or annihilation of particles and entropy in a comoving volume because of the fact that there is no flow into or out of a comoving volume element.

Now, there are different possibilities to relate the three types of quantities.

1. Identification of thermodynamic and effective quantities. The consequence of this identification is: no particle and no entropy creation (annihilation is negative creation) in a comoving volume. Here one can ask the question of whether this could be the root of some problems in cosmology.

2a. Identification of thermodynamic and effective energy density only. This identification seems to be suggested by the weak equivalence principle. Prigogine *et al.* additionally assume that the universe is an adiabatically closed but otherwise open system, meaning $dQ=0$, or

$$(sa^3)^{\cdot}=s_a(na^3)^{\cdot} \tag{5}$$

Entropy creation is not independent of particle creation, but is determined by the latter.

Taking into account that $dQ=0$, equations (2)-(4) allow us to express the effective pressure $\bar{p}$ by thermodynamic quantities and those describing the gravitational field. This relation is not written down here because later we shall formulate all the relations using thermodynamic quantities only.

To close the system of equations, an additional relation is postulated that shall model the particle creation out of the gravitational field:

$$(na^3)^{\cdot}=\alpha H^2 a^3 \tag{6}$$

(α being a parameter greater than zero). The dynamics of a universe filled with dust is ruled by

$$\dot{H}=-\tfrac{3}{2}H^2+\frac{\alpha\kappa M}{6}H \tag{7}$$

(κ is Einstein's constant of gravitation and M is the rest mass of the particles filling the universe). A consequence of this approach is the de Sitter universe is the end of cosmic evolution. But this seems not to be the case realized in nature and, therefore, creation of particles is switched off by hand at some moment in time.

2b. Identification as before, but another ansatz for particle creation (Kasper, 1990). Particle creation is modeled by

$$(na^3)^{\cdot}=3LH^2[\tfrac{2}{3}(1+\tfrac{5}{2}\beta)-\beta L^2H^2](na^3) \tag{8}$$

(β and L parameters). The dynamics is ruled by

$$\dot{H}=-\tfrac{3}{2}H^2[1-\tfrac{2}{3}(1+\tfrac{5}{2}\beta)LH+\beta L^3H^3] \tag{9}$$

A consequence of this is the existence of a quasi-de Sitter stage of cosmic evolution and a smooth approach to the Einstein–de Sitter expansion without a switching off of the particle creation at some finite moment in time.

Turok (1988) has argued that there should have been a string-driven inflation modeled by

$$(\rho a^{\alpha})^{\cdot}-\frac{\beta}{\mu\pi}a^{\alpha}\rho H^3=0 \tag{10}$$

(α and β parameters of order 1, μ string tension). But the relation between "*particle*" number density n and *energy* density ρ and whether the system is adiabatically closed and allows particle creation is not discussed [For further remarks concerning this point see Kasper (1992)].

3. PHENOMENOLOGICAL DESCRIPTION OF MATTER AND HIGHER-ORDER GRAVITY

The general case is an entropy creation independent of particle creation in a comoving volume. It offers also a possibility to consider so-called higher-order gravity from the point of view of a phenomenological description of matter and gravitation (see the remarks at the end of the Introduction). If one wants to do more than model a homogeneous and isotropic universe, one needs a more general approach than that discussed in Section 2. For this reason, we start from a variational principle and follow Bailyn (1980). The variational principle reads

$$\begin{aligned}\delta \int_{\Omega} \{R-2\kappa\rho-2\kappa\phi[(nu^i)_{;i}-\alpha R^2]\}(-\det g_{ik})^{1/2}\, d^4x \\ -2\kappa\delta \int_{\Omega} \lambda[(su^i)_{;i}-\beta kR^2](-\det g_{ik})^{1/2}\, d^4x \\ +\kappa\delta \int_{\Omega} n\psi(g_{ik}u^iu^k+1)(-\det g_{ik})^{1/2}\, d^4x = 0 \qquad (11)\end{aligned}$$

R is the curvature scalar, $\det g_{ik}$ is the determinant of the metric fundamental tensor $\mathbf{g}$ with the components g_{ik}, k is Boltzmann's constant, and a semicolon means covariant derivative defined by the Levi-Civita connection given by the metric $\mathbf{g}$. The u^i are the components of the four-velocity of particles.

ρ shall be the energy density of a simple fluid and, therefore, a function of the particle number density n and the entropy density s. Instead of ρ we shall use the free energy density f given by $f=\rho-sT$. The origin of the integrands in (11), except for $R-2\kappa\rho$, are constraints. The first two model the particle and entropy creation whose source is the "geometrical background" represented by the square of the curvature scalar (α and β, not to be confused with those in Section 2, are dimensionless parameters). Of course, there are other "squared curvature terms," but for simplicity only R^2 is chosen. The *square* of R is chosen (1) so as not to introduce new parameters having a dimension, (2) to end with field equations approaching those of Einstein's theory for weak curvature, and (3) last but not least because the renormalization of the energy-momentum tensor of a quantized field in curved space-time leads to such "squared" curvature terms (for example, Fulling, 1989). ϕ, λ, and ψ are Lagrange multipliers.

Let me stress the point once again the all of what follows is formulated using thermodynamic quantities and not effective ones. For the motivation of this kind of variational principle see the remarks at the end of the Introduction.

Variation is taken with respect to g_{ik}, n, u^i, T, and ϕ, λ, ψ. With the abbrevations E^{ik} and S^{ik} for the components of the Einstein tensor and its traceless part and z for $(\alpha\phi+\beta k\lambda)$, the result is

$$\begin{aligned} A^{ik} := -E^{ik} - \kappa(\rho g^{ik} - n\psi u^i u^k) - \kappa(n\phi_{,r} + s\lambda_{,r}) u^r g^{ik} \\ -4\kappa z S^{ik} + 4\kappa (zR)_{;rm}(g^{im}g^{kr} - g^{mr}g^{ik}) = 0 \end{aligned} \tag{12}$$

$$B := 2\kappa\left[-\frac{\partial f}{\partial n} + (-T + \lambda_{,i}u^i)\frac{\partial s}{\partial n} + \phi_{,i}u^i\right] = 0 \tag{13}$$

$$C_i := 2\kappa(n\psi u_i + n\phi_{,i} + s\lambda_{,i}) = 0 \tag{14}$$

$$D := 2\kappa\left[-\frac{\partial f}{\partial T} - (T - \lambda_{,i}u^i)\frac{\partial s}{\partial T} - s\right] = 0 \tag{15}$$

$$(nu^i)_{;i} = \alpha R^2 \tag{16}$$

$$(su^i)_{;i} = \beta k R^2 \tag{17}$$

$$g_{ik}u^i u^k = -1 \tag{18}$$

($A_{,k}$ means derivative with respect to x^k). Now, take into account that $\partial f/\partial T = s$ and $\partial f/\partial n = (f+p)/n$, then this results in

$$\psi = \frac{\rho + p}{n} \tag{19}$$

$$\lambda_{,i}u^i = T \tag{20}$$

$$\begin{aligned} E^{ik} - \kappa[(\rho+p)u^i u^k + p g^{ik}] \\ + 4\kappa z R S^{ik} - 4\kappa(zR)_{;rm}(g^{im}g^{kr} - g^{mr}g^{ik}) = 0 \end{aligned} \tag{21}$$

The difference between equation (21) and the fourth-order gravity field equation obtained if only R^2 enters the Lagrange density in addition to the usual terms is that we have in (21) zR instead of R only in the last two terms.

Finally, with the abbreviation Δ^r_s for $\delta^r_s + u^r u_s$, the combination $B_{,i} - (C_i/n)_{;k}u^k = 0$ leads to the equation of motion

$$(\rho+p)u_{i;k}u^k + p_{,k}\Delta^k_i + \left[(su^k)_{;k} - \frac{s}{n}(nu^k)_{;k}\right]\Delta^r_i \lambda_{,r} = 0 \tag{22}$$

This system of field equations has some special properties: We do not get rid of the Lagrange multipliers ϕ and λ on the level of differential equations. Their derivatives in the direction of the particle flow are related to thermodynamic quantities such as temperature and chemical potential per particle. Formally, we would obtain the Lagrange multipliers by integration along the streamlines of particles. Then, the Einstein-like equation is an integral equation. The history of the physical system enters this equation. We can always choose such initial conditions that ϕ and λ are equal to zero on a spacelike hypersurface. Near this hypersurface, (21) is then Einstein's equation with small corrections. But far away from this arbitrarily chosen hypersurface, (21) is quite different from the Einstein equation. One of the terms in (21) contains the second derivatives of the curvature scalar. Therefore, (21) is, indeed, a fourth-order equation. It is the consequence of the constraints modeling the creation of particles and entropy. The idea is that such constraints could possibly reflect in a phenomenological way the particle creation we meet in quantum field theory.

Let us now consider some aspects of a homogeneous and isotropic world model with flat three-spaces. For this case, the field equations read as follows:

$$H^2 = \frac{\kappa}{3}\rho + 12\kappa \dot{H} z(\dot{H} + 2H^2) - 24\kappa H[z(\dot{H} + 2H^2)]^{\cdot} \tag{23}$$

$$\dot{\phi} = \frac{\rho + p - sT}{n} \tag{24}$$

$$\dot{\lambda} = T \tag{25}$$

$$\dot{n} + 3Hn = 36\alpha(\dot{H} + 2H^2)^2 \tag{26}$$

$$\dot{s} + 3Hs = 36\beta k(\dot{H} + 2H^2)^2 \tag{27}$$

(an overdot means time derivative). Now, let us suppose that n, s, T, ρ, and p are constant. Then, the relation

$$H^2 = \frac{\kappa}{3}\rho - \kappa(\rho + p) \tag{28}$$

results. The de Sitter universe of Einstein's theory with $p = -\rho$ is a special solution and, in general, a positive energy density asks for a negative pressure, contrary to the result in Prigogine *et al.* (1988). The reason for this seems to be that if one starts from the variational principle (11), also the relation between H and ρ is changed, contrary to the situation in (3). Let us finish this section by considering the universe filled with radiation. Then,

$$\rho = aT^4, \qquad n = bT^3, \qquad s = \tfrac{4}{3}aT^3 \tag{29}$$

(a and b are some constants). Because of the fact that the chemical potential of radiation is equal to zero,

$$\dot{z} = \beta kT \tag{30}$$

Finally,

$$4\kappa H z \dot{R} = -H^2 + \frac{\kappa}{3} aT^4 + 2\kappa(\tfrac{1}{6}R - 2H^2)zR - 4\kappa\beta kTHR \tag{31}$$

and

$$T^2\dot{T} = -HT^3 + \frac{\alpha}{3b} R^2 \tag{32}$$

The entropy density s and number density of photons n are such that $n/s = 3b/4a$ and the constraint equations lead to $4a/3b = \beta k/\alpha$.

For $R = 0$, we obtain the radiation universe of Einstein's theory. $T = H = R = 0$ and z an arbitrary constant are the singular line elements of the system of differential equations. The line $H(t) = R(t) = T(t) = 0$ and $z(t) =$ const is a solution and near this line there are initial conditions for which, for example, $\dot{T}$ becomes arbitrarily large:

$$\dot{T} = -HT + \frac{\alpha}{3b}\frac{R^2}{T^2} \tag{33}$$

($\alpha/3b$ is greater than zero). This is a hint that the Minkowski space-time is unstable. And it could be that we have obtained a model that gives a phenomenological description of the transition from vacuum to a radiation universe caused by vacuum fluctuations.

The system of field equations derived from the variational principle (11) is rather complicated and shall be discussed more extensively elsewhere.

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